Particle Swarm Optimization of High-frequency Transformer

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***Abstract*—A high frequency transformer is a critical component in a dual active bridge converter (DAB) used in a power electronics-based solid state transformer. Operation of a DAB converter requires its transformer to have a specific amount of winding leakage inductance. The demand for high efficiency requires minimization of transformer copper loss and core loss. Furthermore, available window area limits the winding arrangement of transformer. A hybrid of particle swarm optimization (PSO) and differential evolution (DE) is proposed to solve this multi-objective problem with constraints. DE provides diversity to pbest of each particle, which is missing in standard PSO. The differential evolution particle swarm optimization (DEPSO) algorithm is applied to find optimal transformer designs for DAB converters. Results show the DEPSO method is a generic effective way to find optimal high frequency transformer design.**

***Index Terms*—Solid State Transformer, High Frequency Transformer, Particle Swarm Optimization**

I. INTRODUCTION

Power electronic converter-based solid state transformers (SSTs) have become attractive during recent years [1] [2]. An SST has smaller physical profile than a conventional passive transformer does. More importantly, SST provides power flow control at distributive level, such as active power control, reactive power control, harmonic compensation and voltage ride-through compensation, which is demanded by future smart grid. Candidate SST topologies have dual active bridge (DAB) converters (Fig. 1), which contain high frequency transformers to provide both galvanic isolation and leakage inductance for soft switching [1]. This high frequency transformer is critically important for the operation of SSTs. Also, copper loss and core loss need to be minimized to increase overall power efficiency. What is more, a successful design is constrained by the available core area and window area of a given magnetic core. All of these make the transformer design for a DAB converter a complex problem.

Transformer optimal design is a nonlinear multi-dimensional problem with constraints [3], [4]. Evolutionary computation and swarm intelligence methods have the potential in solving this kind of problem. Evolutionary design has been applied to transformer cost optimization [5]. Genetic algorithms is proposed to minimize the weight and the loss of the medium frequency transformers for switch mode power supplies [6]. Particle swarm optimization (PSO) is a population-based stochastic swarm intelligence optimization technique [7]. PSO is capable of finding global optimal points by using unsophisticated agents. PSO has been applied in transformer fault diagnosis [8], [9]. However, PSO lacks diversity in its principles, which can be compensated by differential evolution (DE) [10]. DE gives mutual diversity using mutation and crossover. Thus, Differential Evolution Particle Swarm Optimization (DEPSO), a hybrid of PSO and DE, could obtain better optimization performance [7].

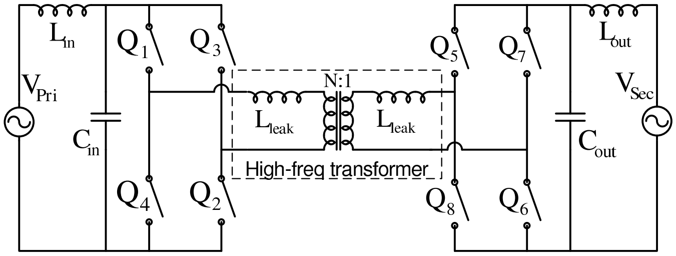

Fig. 1. DAB Converter Schematic

This study presents a DEPSO-based approach to optimize winding configurations for the transformers in DAB converters. The proposed method minimizes power loss and the difference between winding leakage inductance and its design specification. The remainder of this paper is organized as follows. Section II describes the objective functions and constraints of transformer design. Section III describes the DEPSO optimization algorithm. Section IV presents the implementation of the proposed algorithm. Section V discusses the results of a case study. Conclusions are made in Section VI.

II. PROBLEM DESCRIPTION

For simplicity, the type and size of transformer magnetic core are fixed by engineering method [11]. The focus in this study is to find an optimal winding configuration. This section describes the modeling of winding impedance and power loss.

A. Winding Leakage Inductance

A case of winding configuration is shown in Fig. 2. Transformer winding leakage inductance is evaluated according the one-dimensional method introduced in [12]. According to [12], there are three contributions to winding leakage inductance.

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Firstly, leakage inductance due to the gaps between winding portions is frequency independent, which is given by

$$L_g = \frac{\mu_0 N_p^2 l_T g}{b}, \tag{1}$$

where μ_0 is flux magnetic constant, N_p is the number of turns in this portion, l_T is the Mean Turn Length (MTL) of a given core, g is the height of inter-portion gap, and b is the width of winding breadth.

To simplify problem description, only full portions (portions without a half layer of wires) are considered in this study. In this case, leakage inductance due to inter-layer gaps in one winding portion (which is also frequency independent) is

$$L_U = \frac{\mu_0 N_p^2 l_T U}{3b}\left(1 - \frac{1}{2m}\right), \tag{2}$$

where m is the number of whole layers in a portion and U is the total height interlayer gaps.

Leakage inductance due to magnetic wires is frequency dependent. It is the product of its dc component and a frequency ratio. For a full portion, they are

$$L_w = F_L L_{dc}, \tag{3}$$

$$L_{dc} = \frac{\mu_0 m^3 N_l^2 l_T h}{3b}, \tag{4}$$

$$F_L = \frac{3M^{''} + \left(m^2 - 1\right) D^{''}}{m^2 |\alpha^2 h^2|}, \tag{5}$$

$$M'' = Im\{\alpha h \coth\left(\alpha h\right)\}, \tag{6}$$

$$D'' = Im\{2\alpha h \tanh \frac{\alpha h}{2}\}, \tag{7}$$

$$\alpha = \sqrt{\frac{j\omega\mu_0\eta}{\rho}}, \tag{8}$$

$$\eta = \frac{N_l a}{b}, \tag{9}$$

where N_l is the number of turns per layer and h is the height of conductor, ω is the radial frequency of transformer, μ_0 is the vacuum permeability, and a is the breadth of a conductor. $Im\{\}$ is an operator of taking the imaginary part of a complex number. Detailed explanation of (3) – (9) can be found in [5], which is not repeated here.

Note that a winding portion is defined by m.m.f varies between peak and zero. Also note that Fig. 2 only shows half the winding area. Thus total leakage inductance is twice of the value obtained by (3) – (9).

B. Winding Ac Resistance

Ac winding resistance of high frequency transformer is calculated using the orthogonality between skin effect and proximity effect [13]. Skin effect and proximity effect can be described by increasing resistance as frequency increases.

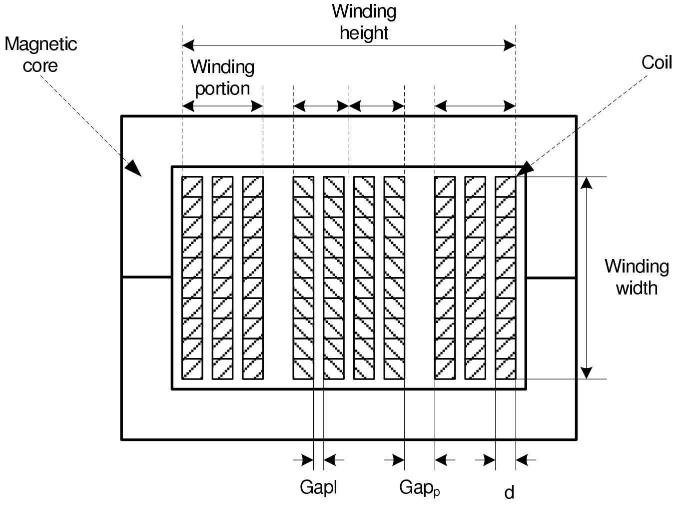


Fig. 2. Transformer Winding Schematic

Ac resistance is the product of dc resistance and a factor representing skin and proximity effects, which is given by

$$k = \frac{\sinh\xi(\eta) + \sin\xi(\eta)}{\cosh\xi(\eta) - \cos\xi(\eta)} + \eta^2\left(2m - 1\right)\frac{\sinh\xi(\eta) - \sin\xi(\eta)}{\cosh\xi(\eta) + \cos\xi(\eta)}, \tag{10}$$

where $\xi(\eta) = \frac{\sqrt{\pi}}{2}\frac{d}{\delta(\eta)}$, $\delta(\eta) = \frac{1}{\sqrt{\pi f \mu_0 \eta/\rho}}$, and d is the diameter of wires. Dc component of winding resistance is

$$R_{dc} = \frac{\rho N l_T}{A}, \tag{11}$$

where ρ is the resistivity of copper, N is total number of turns, and A is wire copper area. Then ac winding resistance is the product of dc resistance and the ratio representing resistance increment due to skin effect and proximity effect.

C. Loss Modeling

Power loss in transformer consists of core loss and copper loss.

Copper loss is

$$P_{cu} = I_{rms,p}^2 R_{ac,p} + I_{rms,s}^2 R_{ac,s}, \tag{12}$$

where $R_{ac,p}$ and $R_{ac,s}$ are the ac resistance of primary and secondary windings, respectively, and $I_{rms,p}$ and $I_{rms,s}$ are the RMS current at primary and secondary windings, respectively.

According to [11], peak flux density is

$$B_{ac} = \frac{V_{rms} \times 10^4}{K_f N f_{sw} A_e}, \tag{13}$$

where V_{rms} is the RMS voltage at primary side, N is the number of turns at primary side, f_{sw} is the switching frequency of DAB converters, A_e is the effective area of a given magnetic core, and K_f is waveform coefficient. Then core loss is given by

$$P_{core} = k f^m B_{ac}^n, \tag{14}$$

where parameters k, m, and n for a given core material can be found in [11] or from a manufacturer's datasheet.

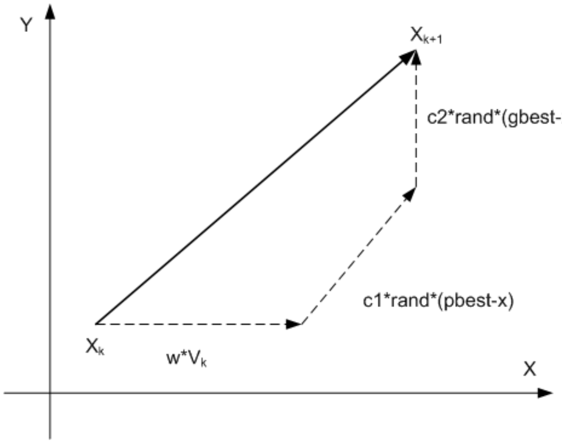


Fig. 3. PSO Position Update

D. Design Constraints

Winding configuration of a transformer is limited by available window area of a given magnetic core. The width of winding is

$$Width = N_l d. \tag{15}$$

The height of winding is

$$Height = \sum^{n} m_i d + nG_p + \left(\sum^{n} m_i - 1 \right) G_l, \tag{16}$$

where d is the diameter of magnetic wires, n is the number of winding portions, G_p is the gap between winding portions, and G_l is the gap between layers in a winding portion.

The width and height of winding coil should be less than those specified in the magnetic core datasheet

$$\begin{cases} Width < WindowWidth, \\ Height < WindowHeight, \end{cases} \tag{17}$$

III. DEPSO ALGORITHM

A. Particle Swarm Optimization

PSO is a population based optimization algorithm, inspired by flock of birds or school of fish movements [7]. In PSO, a population of particles flies through an unknown space with velocity updated by inertia, cognition and social interaction [7]. Velocity and position of each particle is updated by the following equations.

$$\begin{aligned} v_i(k) = &\omega \cdot v_i(k) + c_1 \cdot rand() \cdot (pbest_i - x_i(k)) + \\ &c_2 \cdot rand() \cdot (gbest_i - x_i(k)), \end{aligned} \tag{18}$$

$$x_i(k+1) = v_i(k) + x_i(k), \tag{19}$$

where x_i is the position of the i-th particle in variable space, v_i is the velocity of the i-th particle, ω is particle inertia, c_1 is cognitive acceleration constant, c_2 is social acceleration constant, pbest is particle's best known position, gbest is the best known position found by all particles, and rand() means a random number between 0 and 1. The scheme of position update is illustrated in Fig. 3.

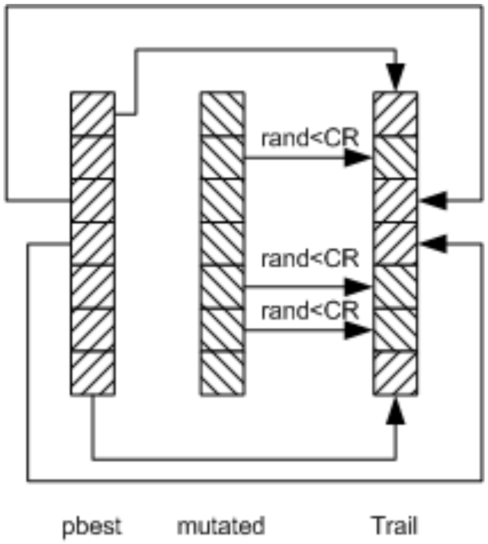


Fig. 4. DE Crossover

B. Differential Evolution

DE is a population based evolutionary algorithm introduced by Storn and Price [10]. Compared to GA, DE can represent real number problems, which makes it possible to integrate with PSO. DE defines a set of crossover and mutation operators in real number space. Mutation is given by

$$\Delta_i(k) = \frac{1}{2} \left(x_i(k) + x_a(k) - x_b(k) - x_c(k) \right), \tag{20}$$

where a, b, and c are parents used to generate an off-spring for the i-th particle. And crossover is given by

$$v_{ij}(k) = \begin{cases} x_{ij}(k) + \Delta_{ij}(k), & \text{if } rand() \leq CR \\ x_{ij}(k), & \text{if otherwise,} \end{cases} \tag{21}$$

where CR is crossover rate (Fig. 4). Parents and offstrings compete with each other according to a fitness function so that those with better fitness are select to form a new generation and the rest candidates are discarded.

C. DEPSO

DEPSO is a hybrid of DE and PSO. DE provides the diversity that lacks in PSO while retaining the search capability of PSO [7]. The mechanism of DEPSO is: after position update in every normal PSO iteration, DE operator is applied to every pbest of each particle. In this way, the DEPSO algorithm is shown in Fig. 5.

IV. IMPLEMENTATION OF DEPSO ALGORITHM

A. Fitness Function and Constraints

For leakage inductance, fitness metric is the relative error of calculated leakage inductance L to expected leakage inductance L^*.

$$F_L = \frac{|L - L^*|}{L^*}. \tag{22}$$

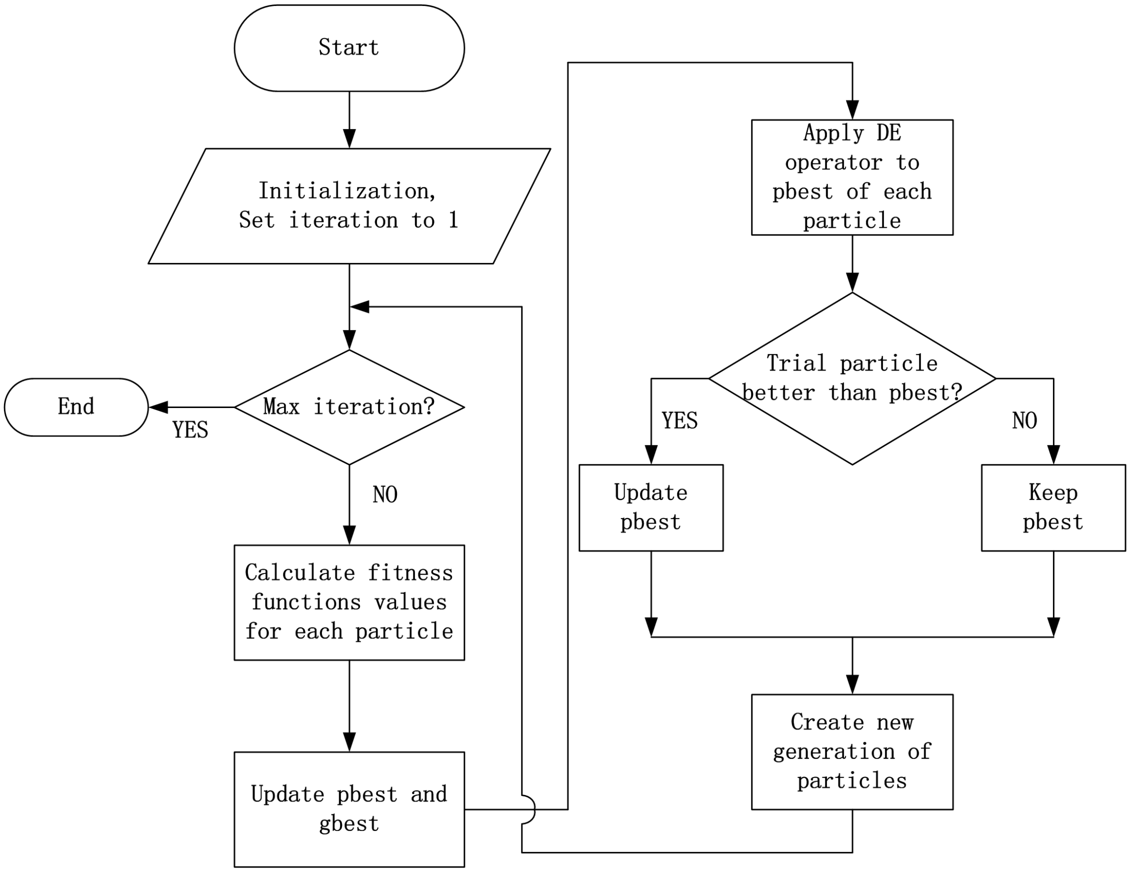


Fig. 5. DEPSO flowchart

A diode-like function is used to normalize power loss and provide penalty to high power loss.

$$F_P = 0.3 \exp\left(\frac{P_{core} + P_{cu}}{0.65P^*}\right), \qquad (23)$$

where P^* is 7 Watts.

Using weighted aggregation, the two objectives can be combined into one fitness function.

$$F = w_1 F_L + w_2 F_P, \qquad (24)$$

where $w_1 = 0.5$ and $w_2 = 0.5$. The optimization target is searching for parameters to minimize fitness function F.

It is necessary to impose the design constraints described in (17). After PSO update and DE operation, each new solution is checked to determine whether the constraints in (17) is satisfied. If not, the new solution is reset to the current pbest position.

B. Particle and Search Space Definition

In this study, the shape and size of the magnetic core, as well as turns ratio, are fixed. The optimizing algorithm only searches for an optimal winding configuration. A particle contains a vector of parameters to be optimized: diameter of magnetic wire, number of wires in parallel for one turn, number of turns in the primary winding, number of winding portions, number of layers in one winding portion, thickness of the gap between winding portions, and thickness of gap between layers in a winding portion. Parameter search space, which is given in Table I, is defined using common sense and experience.

It is obviously that the parameters in Table I are all integers. Therefore, Integer PSO is used in this work [7]. The parameters of all particles are rounded off to the nearest integers.

C. DEPSO Parameters

In PSO, linearly decreasing weight inertia can cover both early exploration and final exploitation. Weight inertia is defined as

$$\omega = 0.8 - 0.4 \frac{iteration}{max_{iteration}}. \qquad (25)$$

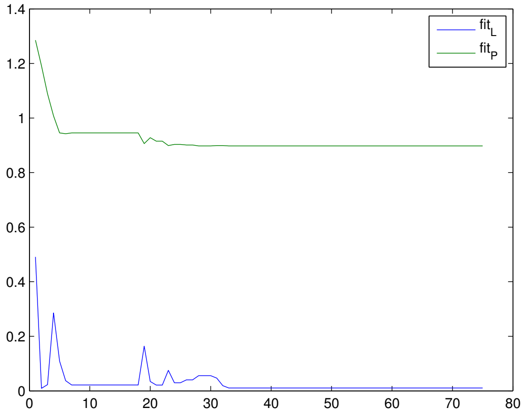
Individual and social acceleration constants are both set to 2.0 [7]. Maximum velocity is set to one-half of the entire search space for each dimension. The number of particles is set to 50 and maximum number of iterations is set to 75. For DE operator, crossover rate is set to 0.5. And the number of parents is set to 4 for mutation.

V. CASE STUDIES AND RESULTS

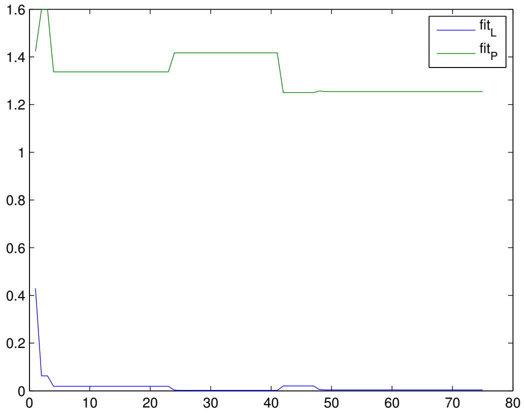
One design case is a transformer for a 200 W 208V-to-120V ac-ac dab converter [1]. The turns-ratio is 1.73:1 and the frequency is 5 kHz. It uses an ETD-59 ferrite magnetic core.

TABLE I
SEARCH SPACE DEFINITION

wire gauge	number of primary turns	number of portions	layers per pri portion	layers per sec portion	layers of NOMEX	layers of Epoxy film
14,16,18,20,22,24	25 – 500	1,2,4,6,8,10	1 – 20	1 – 20	1 – 10	1 – 20



(a) DEPSO



(b) PSO

Fig. 6. Fitness metrics over search process

Target leakage inductance is 900 μH respect to the low voltage side. The searching process (variation of fitness function over iterations) is shown in Fig. 6(a) for DEPSO and Fig. 6(b) for conventional PSO, respectively. Three best transformer designs by DEPSO are given in Table II after 20 trials. Figure 7 compares leakage inductance error and power loss of three design methods: DEPSO, conventional PSO, and engineering area product method. It is shown that the diversity introduced in DEPSO provides better fine tuning over both PSO and hand design.

VI. DISCUSSION AND CONCLUSION

An optimization method is presented to address the high frequency transformer design problem for DAB converters in SSTs. The DEPSO method integrates the searching capability of PSO and diversity of DE. Fitness functions are evaluated by 1-D winding impedance analytical model and engineering power loss estimation method. DE operator is applied in every iteration to the pbest of particles in the population. The DEPSO optimizing algorithm is applied to find optimal designs for transformers in SSTs. Simulation results show this algorithm is an effective and generic method for transformer optimization.

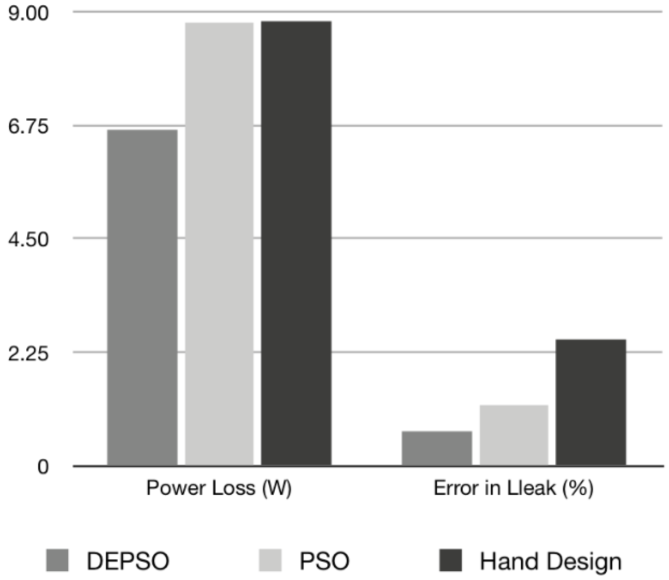


Fig. 7. Comparison among DEPSO, conventional PSO, and hand design

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REFERENCES

[1] H. Qin and J. W. Kimball, “Ac-ac dual active bridge converter for solid state transformer,” in *Energy Conversion Congress and Exposition, 2009. ECCE. IEEE*, 2009, pp. 3039–3044.

[2] T. Zhao, G. Wang, J. Zeng, D. S., B. S., and A. Q. Huang, “Voltage and power balance control for a cascaded multilevel solid state transformer,” in *Applied Power Electronics Conference and Exposition, 2010. APEC ’2010. Twenty-Fifth Annual IEEE*, 2010, pp. 761–767.

[3] E. Amoiralis, M. Tsili, and A. Kladas, “Transformer design and optimization: A literature survey,” *Power Delivery, IEEE Transactions on*, vol. 24, no. 4, pp. 1999 –2024, oct. 2009.

[4] G. Gu and Q. Li, “Optimization design of transformer used in switching power supply with high power density,” in *Power Electronics and Motion Control Conference, 2009. IPEMC ’09. IEEE 6th International*, may. 2009, pp. 1250 –1253.

TABLE II
OPTIMAL TRANSFORMER DESIGNS FOR CASE 1

primary wire	secondary wire	number of primary turns	number of portions	layers per pri portion	layers per sec portion	layers of NOMEX	layers of Epoxy film	L_{leak} error	power loss
1 gauge 20	1 gauge 18	169	1	4	4	3	3	0.69 %	6.66 W
1 gauge 20	3 gauge 22	211	1	3	4	6	2	0.92 %	5.94 W
1 gauge 22	1 gauge 20	116	1	2	7	3	3	0.15 %	7.63 W

[5] E. Amoiralis, P. Georgilakis, M. Tsili, and A. Kladas, “Global transformer optimization method using evolutionary design and numerical field computation,” *Magnetics, IEEE Transactions on*, vol. 45, no. 3, pp. 1720 –1723, mar. 2009.

[6] C. Versele, O. Deblecker, and J. Lobry, “Multiobjective optimal design of transformers for isolated switch mode power supplies,” in *Power Electronics Electrical Drives Automation and Motion (SPEEDAM), 2010 International Symposium on*, jun. 2010, pp. 1687 –1692.

[7] Y. del Valle, G. K. Venayagamoorthy, S. Mohagheghi, J. C. Hernandez, and R. G. Harley, “Particle swarm optimization: Basic concepts, variants and applications in power systems,” *Evolutionary Computation, IEEE Transactions on*, vol. 12, no. 2, pp. 171–195, 2008.

[8] W. Niu, L. Xu, and J. Wu, “The application of particle swarm optimization-based rbf neural network in fault diagnosis of power transformer,” in *Computer Science and Information Technology, 2009. ICCSIT 2009. 2nd IEEE International Conference on*, aug. 2009, pp. 534 –536.

[9] Z. Shi, Y. Li, Y. Song, and T. Yu, “Fault diagnosis of transformer based on quantum-behaved particle swarm optimization-based least squares support vector machines,” in *Information Engineering and Computer Science, 2009. ICIECS 2009. International Conference on*, dec. 2009, pp. 1 –4.

[10] R. Storn and K. Price, “Differential evolution a simple and efficient heuristic for global optimization over continuous spaces,” *Journal of Gloabl Optimization*, vol. 11, no. 4, pp. 341–359, 1997.

[11] C. W. T. McLyman, *Transformer and Inductor Design Handbook*. CRC, 2004.

[12] P. L. Dowell, “Effects of eddy currents transformer windings,” *Proceedings of IEE*, vol. 113, no. 8, pp. 1387–1394, 1966.

[13] J. A. Ferreira, “Improved analytical modeling of conductive losses in magnetic components,” *Power Electronics, IEEE Transactions on*, vol. 9, no. 1, pp. 127–131, 1994.